Infiltration processes in cohesionless soils

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Abstract

In this paper, a thermodynamically consistent four-phase continuum model in the framework of the mixture theory is presented describing infiltration processes of suspensions in cohesionless granular material. The paper focuses on the distinct form of the constitutive relation for the volume production term of the fluidized particles and its consequences on the infiltration process.

To this end a constitutive equation describing infiltration phenomena is proposed which includes only one material parameter. Therefore we study numerically a boundary value problem, which is characterized by a homogeneous field of the hydraulic gradient in the reference configuration at the time $t_0 = 0$. Infiltration is affecting the distribution of the hydraulic properties and illustrates the consequences of the proposed constitutive equation for specific parameter choices. Furthermore it is shown how the material parameter can be estimated without explicit numerical calculations.

1 Introduction

Infiltration processes are characterized by seepage flow of a complex fluid through a porous medium and a possible deposition process of the fines of the suspension. Complex fluids are mixtures of a liquid - mostly water - and small particles (fines) moving with the fluid. According to the necessities of the investigated infiltration process the velocity of the fines may be assumed identical to the fluid velocity.

The deposition of fines in the pore space of the porous medium results in considerable changes of the material properties of the porous medium. On the one hand, mechanical properties are changed. The structural stiffness is increased due to a reduction of pore space and additional material strength coming from the deposited fines - being part of the skeleton after deposition. On the other hand, hydraulic properties like the hydraulic conductivity and the effective viscosity of the particle-laden fluid are modified. Macroscopically, the reduced pore space leads to lower permeabilities as well as to a change of effective viscosities due to the phase change of the former fluidized fines.

Deposition is triggered/controlled in two ways: Hydraulic reasons trigger the process whereas geometrical properties on the microscale control the process. The geometrical requirements for a decomposition process can readily be taken e.g. from Terzaghi's filter laws [15] indicating that some of the fines of the particle-laden fluid cannot pass the constriction sizes of the porous medium. Once these requirements are met, larger deposition rates are achieved by a higher hydraulic gradient. Additionally taking sedimentation effects into account - which is not the case in the proposed model - also gravity forces in conjunction with a rather low hydraulic gradient increase the deposition rate.

Depending on the initial conditions of the the porous medium and the complex fluid (pore constriction size distribution on the microscopical scale of the porous material, hydraulic gradient) there are basically two kinds of infiltration processes which should be distinguished. A steady going one, where deposition of fines takes place rather slowly due to an initially big pore space and a rather low hydraulic gradient and secondly a rapid infiltration which is caused by a high hydraulic gradient and initially small constriction sizes leading to a highly localized clogging effect. Clogging effects during infiltration is very well known in various applications. Mainly, we distinguish between so-called external and internal filter cakes due to clogging phenomena, cf. Figure 5.

Besides various possible applications for the proposed model, such as hardening of soils in mines or land improvements, the authors' main focus is on the field of mechanized tunneling. Due to technical reasons during a typical excavation process of a tunnel lining, a gap is opening between the lining and the surrounding soil. This gap contributes significantly to a change in the mechanical parameters of loose soil, which leads to unwanted subsidence at the surface. Hence, it is desirable to close the gap created by the system of tubbing. To this end the so-called backfilled grout mortar is injected. During the tunneling process distinct material properties of the backfilled grout mortar are needed. First, the mortar should have a good flowability to ensure mechanical processing and transporting of the mortar through the pipelines. As soon as the mortar is in its final position, a rapid hardening is required, which allows to reproduce the primary stress state of the surrounding soil, so that surface subsidence are minimized. The respective strength can be achieved by additives, which cause a hardening process of the mortar. A cheaper and environmentally more friendly approach is to dispense largely with cement portions and to drain the mortar via a high-pressure injection process instead. During this procedure a consolidation process of the mortar takes place, which causes a transport of the mortar's pore fluid into the surrounding soil, so that the mortar is transformed into a solid, gap-filling structure.

2 Governing equations of infiltration processes

Fig. 1 sketches the characteristic micro-structure of a fluid-saturated granular media, which we investigate in the current contribution. The principal material properties are caused by the complex pore fluid which itself is a mixture of a pore liquid and fine-grained particles. Due to drag forces, the complex fluid, i.e. the suspension, could be transported through the porous skeleton or, if the constriction sizes of the pore spaces are too small, the fines are blocked in the pore constriction sizes. Obviously, this local blocking leads to an evolution of the hydromechanical properties of the material. In this contribution, we propose a macroscopic model which is able to predict the evolution of the intrinsic permeability and porosity of the skeleton and therefore could be used for predicting numerically infiltration processes.

In contrast to extensively-discussed poroelastic models describing the hydromechanical behaviour of soils, cf. Biot [1], Coussy [5], we desire the hydraulic infiltration model by the thermodynamically-consistent Theory of Porous Media (TPM) [1, 2, 3, 6, 8], which could be regarded as an extension of the mixture theory, cf. Truesdell [16].



Figure 1: Microscale and REV of a fully-saturated soil and the corresponding four-phase continuum model

Since the fluid is a suspension, two constituents $\varphi^{\mathfrak{a}}$ and $\varphi^{\mathfrak{f}}$ have been introduced to describe its hydraulic behaviour in further detail. The evolution of the fabric of the solid skeleton is modelled by a stable fabric $\varphi^{\mathfrak{sn}}$ and the fines $\varphi^{\mathfrak{sa}}$, which are blocked in the pore space. Thus a liquid-solid phase transition process is taken into account. Related to the current conditions fines can either behave fluid-like (described by constituent $\varphi^{\mathfrak{a}}$) or solid-like, (described by constituent $\varphi^{\mathfrak{sa}}$). This phase transition process will be taken into account in the present model by a mass/volume production term in the corresponding partial mass/volume balances of the constituents $\varphi^{\mathfrak{a}}$ and $\varphi^{\mathfrak{sa}}$, cf. Eq. (10). Altogether, we observe four participating constituents φ^{α} with $\alpha = \{\mathfrak{f}, \mathfrak{a}, \mathfrak{sn}, \mathfrak{sa}\}$, cf Fig. 1.

The macroscopic modelling idea i.e. the split in stable and unstable constituents, traces back to ideas of Vardoulakis [17] and Steeb & Diebels [13] describing the evolution of the fabric in the context of internal erosion. Nevertheless, the specific constitutive relation, which are the driving forces for the evolution of the fabric has not been investigated for infiltration phenomena. If the concentration of the fines is low and if the density contrast between the fines and the pore fluid is not too high, we observe that the velocity of the fines and of the pore fluid is almost identical, i.e. $v_{\mathfrak{a}} = v_{\mathfrak{f}}$. In a certain Representative Elementary Volume (REV) with the volume dv, cf. Fig. 1, the volume fractions of the single constituents φ^{α} are defined as

$$n^{\alpha} = \frac{\mathrm{d}v^{\alpha}}{\mathrm{d}v}.\tag{1}$$

The partial density ρ^{α} is defined as the ratio between the mass dm^{α} of the phase φ^{α} with respect to the total volume dv of the REV, which leads to a relation between partial densities ρ^{α} and effective densities $\rho^{\alpha R}$, the latter one being the ratio between dm^{α} and the actual volume of the phase dv^{α}

$$\rho^{\alpha} = \frac{\mathrm{d}m^{\alpha}}{\mathrm{d}v} = \frac{\mathrm{d}m^{\alpha}}{\mathrm{d}v^{\alpha}} \frac{\mathrm{d}v^{\alpha}}{\mathrm{d}v} = \rho^{\alpha R} n^{\alpha}.$$
(2)

As the 4-phase model is an extension of a previously discussed 3-phase model for application in internal erosion, cf. [13], we briefly sketch the modelling framework. Due to the so-called saturation condition

$$\sum_{\alpha} n^{\alpha} = 1 \quad \rightsquigarrow \quad n^{\alpha} \le 1 \quad \rightsquigarrow \quad \rho^{\alpha} \le \rho^{\alpha R}, \tag{3}$$

the value of the partial density is always smaller than the value of the effective one. The main part of the proposed model is the mass balance of the four phases. Following Ehlers & Bluhm [8], de Boer [6] and Steeb [12], the local form of the partial balance of mass is given as

$$(\rho^{\alpha})'_{\alpha} + \rho^{\alpha} \operatorname{div} \boldsymbol{v}_{\alpha} = \hat{\rho}^{\alpha}, \tag{4}$$

whereas a constraint for the mixture mass production $\sum_{\alpha} \hat{\rho}^{\alpha} = 0$ for thermodynamically closed systems guarantees that there is no mass production in the total mixture $\varphi = \bigcup_{\alpha} \varphi^{\alpha}$, being the assembly of all single phases. Since the partial density is given in a spatial description, the material time derivative $(\rho^{\alpha})'_{\alpha}$ is expressed by the partial and the convective time derivative

$$(\rho^{\alpha})_{\alpha}' = \frac{\partial \rho^{\alpha}}{\partial t} + \operatorname{grad} \rho^{\alpha} \cdot \boldsymbol{v}_{\alpha} = \partial_t \left(\rho^{\alpha}\right) + \operatorname{grad} \rho^{\alpha} \cdot \boldsymbol{v}_{\alpha}.$$
 (5)

Furthermore, we assume that the constituents $\varphi^{\mathfrak{a}}$, $\varphi^{\mathfrak{sa}}$ the fluid constituent $\varphi^{\mathfrak{f}}$ and the solid skeleton $\varphi^{\mathfrak{sn}}$ are materially incompressible. Note that the evolution of the porosity $\phi = 1 - n^{\mathfrak{s}}$ in a certain REV is then described through the deposition process of fines $n^{\mathfrak{sa}}$, i.e. $\phi = 1 - n^{\mathfrak{s}} = 1 - n^{\mathfrak{sn}} - n^{\mathfrak{sa}}$. It is obvious, that the partial density of the solid skeleton is also not a constant

parameter as it depends on the porosity, i.e. $\rho^{\mathfrak{s}} = (1 - \phi) \rho^{\mathfrak{s}R}$. Therewith we can reformulate the mass balance into

$$\partial_t \left(\rho^\alpha \right) + \operatorname{div}(\rho^\alpha \, \boldsymbol{v}_\alpha) = \hat{\rho}^\alpha. \tag{6}$$

Applying the definition of the partial density Eq. (2) and constant effective densities $\rho^{\alpha R} = \rho_0^{\alpha R}$, we obtain

$$\partial_t \left(n^\alpha \, \rho^{\alpha R} \right) + \operatorname{div}(n^\alpha \, \rho^{\alpha R} \, \boldsymbol{v}_\alpha) = \hat{\rho}^\alpha = \hat{n}^\alpha \rho^{\alpha R}. \tag{7}$$

Note, that the partial density exchange rate $\hat{\rho}^{\alpha}$ is split into a part depending on the effective density and a volume-driven exchange rate \hat{n}^{α} . If the effective density is constant, which will be the case for a material incompressible constituent $\varphi^{\alpha R}$, the partial balance of mass is reduced to a partial volume balance

$$\partial_t \left(n^\alpha \right) + \operatorname{div}(n^\alpha \boldsymbol{v}_\alpha) = \hat{n}^\alpha. \tag{8}$$

In the second part of this paper we will mainly discuss the influence of the constitutively determined rate of volumetric mass exchange $\hat{n}^{\alpha}[1/s]$ in more detail. Before we formulate the specific mass balances we discuss some of the assumptions, which we introduce to simplify the hydromechanically-driven infiltration process. Obviously, the attached fines and the primary fabric move with the same solid velocity $\boldsymbol{v}_s = \boldsymbol{v}_{sn} = \boldsymbol{v}_{sa}$. Furthermore, we state that there is no relative movement between fluid and fluidized particles, which can be written as $\boldsymbol{v}_{\mathfrak{l}} = \boldsymbol{v}_{\mathfrak{a}} = \boldsymbol{v}_{\mathfrak{f}}$. Hence, we introduce the relative fluid velocity with respect to the solid phase, i.e. the seepage velocity $\boldsymbol{w}_{\mathfrak{f}}$, for the purpose of materially objective constitutive equations $\boldsymbol{w}_{\mathfrak{f}} = \boldsymbol{w}_{\mathfrak{a}} = \boldsymbol{v}_{\mathfrak{f}} - \boldsymbol{v}_{\mathfrak{s}}$. In a next step, we introduce new field variables that are more suitable for physical interpretations. Thus, we introduce the porosity field $\phi(\boldsymbol{x},t)$, the amount of attached fines $a(\boldsymbol{x},t)$ which is related to the solid phase, and, the concentration $c(\boldsymbol{x},t)$ of the fluidized fines in the liquid suspension.

$$\phi = n^{\dagger} + n^{\mathfrak{a}},$$

$$n^{\mathfrak{s}} = n^{\mathfrak{sn}} + n^{\mathfrak{sa}} = 1 - \phi,$$

$$a = \frac{n^{\mathfrak{sa}}}{n^{\mathfrak{s}}},$$

$$c = \frac{n^{\mathfrak{a}}}{\phi}.$$
(9)

2.1 Mass balances

As already mentioned in the previous sections, the transformation of state of the fluidized fines into an attached particle is modelled via the rate of mass exchange \hat{n}^{α} between the phases $\varphi^{\mathfrak{sa}}$ and $\varphi^{\mathfrak{a}}$. The production of attached fines is equal to the reduction of fluidized particles in the REV. In the present paper, we focus on the hydraulic influence of this infiltration processes. An evolution of mechanical properties like the stiffness of the porous skeleton is out of the scope. Thus the deformation process of the skeleton will be neglected ($u_{\mathfrak{s}} = v_{\mathfrak{s}} = 0$) leading to

$$\hat{n}^{\mathfrak{a}} = -\hat{n}^{\mathfrak{sa}} =: -\hat{n}^{\mathfrak{s}}. \tag{10}$$

The governing equations of the infiltration process are stated in the following. We start with the balance of mass of the mixture which is the sum of all partial mass balances Eq. (8). Transforming it using the definitions given above and the saturation constraint Eq. (3)

$$\underbrace{\partial_t \underbrace{(n^{\dagger} + n^{\mathfrak{a}} + n^{\mathfrak{sn}} + n^{\mathfrak{sn}})}_{0}}_{0} + \sum_{\alpha} \operatorname{div}(n^{\alpha} \boldsymbol{v}_{\alpha}) = \underbrace{\sum_{\alpha} \hat{n}^{\alpha}}_{0}, \quad (11)$$

leads finally to

$$\operatorname{div} \boldsymbol{v}_{\mathfrak{s}} + \operatorname{div} \boldsymbol{q} = 0. \tag{12}$$

Note, that the vanishing right hand side in Eq. (11) is a result of vanishing mass productions $\hat{\rho}^{\mathfrak{f}}$ and $\hat{\rho}^{\mathfrak{sn}}$ and identical effective densities $\rho^{\alpha R}$ of the phases $\varphi^{\mathfrak{sa}}$ and $\varphi^{\mathfrak{a}}$. In the balance of mass of the mixture, the so-called filter velocity $\boldsymbol{q} = \phi \, \boldsymbol{w}_{\mathfrak{f}}$, has been introduced. The volume balance of porosity, i.e. of the liquid suspension $\varphi^{\mathfrak{l}}$ is obtained by summing up the volume balance of the pore liquid $\varphi^{\mathfrak{f}}$ and the volume balance of the fluidized fines $\varphi^{\mathfrak{a}}$

$$\partial_t \phi + \operatorname{div} \boldsymbol{q} = \hat{n}^{\mathfrak{a}}.$$
(13)

Summing up the balance of volume of the primary fabric $\varphi^{\mathfrak{sn}}$ and the balance of attached fines $\varphi^{\mathfrak{sa}}$, we obtain an evolution equation for the porosity

$$\partial_t \phi = \hat{n}^{\mathfrak{a}}.\tag{14}$$

The balance of volume of the attached fines $\varphi^{\mathfrak{sa}}$ is similarly collapsing to an evolution equation for the amount of fines $a(\mathbf{x}, t)$

$$\partial_t(a\phi) - \partial_t(a) = \hat{n}^{\mathfrak{a}}.$$
(15)

Later we will show, that both ordinary differential equations Eq. (14),(15) can be solved locally. The balance of volume of the fluidized fines $\varphi^{\mathfrak{a}}$ in the suspension reads

$$\partial_t(c\,\phi) + \operatorname{div}(c\,\boldsymbol{q}) = \hat{n}^{\mathfrak{a}}.\tag{16}$$

Summing up all local partial mass balance of the constituents φ^{α} with $\alpha = \{\mathfrak{f}, \mathfrak{a}, \mathfrak{sa}, \mathfrak{sn}\}$ we obtain the continuity equation of the mixture φ

$$\operatorname{div} \boldsymbol{q} = 0. \tag{17}$$

Note, we are able to reduce the set of microscale quantities from four to three (ϕ, a, c) . The integration of the mass balance of the rigid primary fabric leads to

$$n^{\mathfrak{sn}} = n_0^{\mathfrak{sn}} \left(1 - \operatorname{div} \boldsymbol{u}_{\mathfrak{s}} \right) = n_0^{\mathfrak{sn}},\tag{18}$$

in which the partial volume $n^{\mathfrak{sn}}$ can be expressed using the porosity ϕ and the amount of attached fines a

$$n^{\mathfrak{sn}} = (1 - \phi)(1 - a), \quad \text{and} \quad n_0^{\mathfrak{sn}} = (1 - \phi_0)(1 - a_0), \quad (19)$$

leading to

$$\phi = 1 - \frac{(1 - a_0)(1 - \phi_0)}{(1 - a)},\tag{20}$$

which can be used to eliminate one dependent variable from the set of equations

$$a = a(\phi) = \frac{a_0(1 - \phi_0) + \phi_0 - \phi}{(1 - \phi)}.$$
(21)

The quasi-static balance of momentum of the fluid without volume forces, the balance of mass of the fluid and constitutive assumptions reveal the well-known Darcy relationship between the gradient of the fluid pressure and the filter velocity, cf. Ehlers & Bluhm [8]. Here η^{IR} is the effective dynamic viscosity of the particle-laden suspension depending on the amount of fluidized particles $c(\boldsymbol{x}, t)$.

$$\boldsymbol{q} = -\frac{k^{\mathfrak{s}}(\phi)}{\eta^{\mathsf{IR}}(c)} \operatorname{grad} p.$$
(22)

 $k^{\mathfrak{s}}(\phi)$ is the intrinsic permeability of the porous fabric depending on the porosity $\phi(\boldsymbol{x}, t)$. For the considered case of cohesionless granular media, the intrinsic permeability $k^{\mathfrak{s}}$ [m²] could be related to the grain size distribution function of the particle ensemble. Here, we calculate the permeability at the initial state $k_0^{\mathfrak{s}} = k^{\mathfrak{s}}(\boldsymbol{x}, t_0)$ with the Kozeny-Carman equation, cf. Carrier [4]. The evolution of permeability $k^{\mathfrak{s}}(\boldsymbol{x}, t)$ is related to the evolution of the porosity $\phi(\boldsymbol{x}, t)$

$$k^{\mathfrak{s}}(\phi) = k_0^{\mathfrak{s}} \left[\frac{\phi^3}{(1-\phi)^2} \right] \left[\frac{(1-\phi_0)^2}{\phi_0^3} \right] \quad \text{and} \quad k_0^{\mathfrak{s}} = \frac{1}{C_1} \frac{\phi^3}{(1-\phi)^2} D_{eq}^2.$$
(23)

 C_1 is the so-called Kozeny-Carman constant, cf. Irmay [11] or Ergun [10]. D_{eq} is the equivalent particle diameter of the granular material. Einstein [9] proposed the evolution of the effective dynamic viscosity of a suspension. From an initial dynamic viscosity of a liquid η^{fR} , it could be calculated as a function of concentration

$$\eta^{\mathfrak{l}R}(c) = \eta^{\mathfrak{f}R}\left(1 + \frac{5}{2}c\right). \tag{24}$$

It has to be noted that Eq. (24) has initially been proposed for dilute suspensions. For more sophisticated viscosity corrections, e.g. for dense suspensions, we refer to [7].

Thus, the following set of equations is remaining and is formulating the Initially Boundary Value Problem (IBVP) of infiltration:

$$\operatorname{div} \left[\frac{k^{\mathfrak{s}}(\phi)}{\eta^{\mathfrak{f}}(c)} \operatorname{grad} p \right] = 0, \qquad \forall \boldsymbol{x} \in \mathcal{B} \times T$$
$$\partial_t(c\,\phi) + \operatorname{div} \left[c \, \frac{k^{\mathfrak{s}}(\phi)}{\eta^{\mathfrak{f}}(c)} \operatorname{grad} p \right] = \hat{n}^{\mathfrak{a}}, \qquad \forall \boldsymbol{x} \in \mathcal{B} \times T$$

with boundary conditions for the flux \boldsymbol{q} at the Neumann boundary Γ_N and the pressure p at the Dirichlet boundary Γ_D

$$q = \boldsymbol{q} \cdot \boldsymbol{n} = \overline{q}, \qquad \forall \boldsymbol{x} \in \Gamma_N \times T$$
$$p = \overline{p}. \qquad \forall \boldsymbol{x} \in \Gamma_D \times T$$

The coupled nonlinear IBVP could be solved by numerical methods. Here, we use Galerkin-type finite element schemes, described in Steeb [14]. The

primary variables are $\mathcal{P} = \{p, c\}$. The IBVP is completed, in combination with the evolution equation $\partial_t \phi = \hat{n}^{\mathfrak{a}}$ and Eq. (21) for $a(\boldsymbol{x}, t)$ and an appropriate constitutive equation for the rate of mass/volume production $\hat{n}^{\mathfrak{a}}$. The form of the latter equation will be focussed in the next section.

3 Constitutive equation for mass exchange

To close the IBVP, it is necessary to find a formulation for the volume exchange term, the so-called production term \hat{n}^{a} . The structure of the term is already known from previous publications, cf. [13, 12, 17]. In the context of the mathematical formulation of general internal remodelling phenomena described within the mixture theory by a mass production term, Steeb and Diebels [13] pointed out, that a thermodynamically consistent form of the constitutive equation for mass production reads

$$\hat{n}^{\mathfrak{a}} \propto |\boldsymbol{q}| \,.$$
 (25)

Physically this expression is also evident. The higher the flow in a REV, the more particles per time unit will pass the REV. Hence the probability of infiltration of individual particles or swarms of particles is higher. If we consider the limiting case of a vanishing velocity (q = 0), then also the value of the production term disappears. This fact can not be motivated physically, because for such a case of a vanishing fluid velocity one could observe sedimentation of fluidized particles. At least this model focuses on infiltration phenomena and sedimentation is not taken into account. However, it is readily possible to consider a relative movement of fines $w_a \neq w_f$ and an additional expression in the production term to describe these phenomena.

As already described, the probability of infiltration increases with an increasing number of particles, which pass through the REV per time unit. This may be a result from a higher flow rate or of a higher concentration of fluidized particles in the fluid. Following the above argumentation, the probability of infiltration increases with increasing concentration of fines c. Therefore, the production term may be written as $\hat{n}^{\mathfrak{a}} \propto c |\mathbf{q}|$. In this case for c = 0, $\hat{n}^{\mathfrak{a}} = 0$ is obtained. This relationship illustrates that infiltration can only take place if there are fluidized particles in the considered area.

To complete the expression for the production term a second parameter k is added. This parameter is a scalar value, which has to be determined by

experimental investigations. Finally the complete production term can be written as

$$\hat{n}^{\mathfrak{a}} = -k c |\boldsymbol{q}|. \tag{26}$$

In the following section a numerical example will be analysed to illustrate the complete set of equations. Subsequently, the recently introduced parameter k will be discussed on the basis of the results of the numerical example.

4 Analysis of a 1-dim infiltration problem

The numerical solution of the IBVP will be explained in a 1-dim setting. The investigated volume is fully saturated with the four previously described phases φ^{α} . Dirichlet boundary conditions for the pressure \bar{p} are chosen for the boundaries at x = 0 m and x = L. This is done in a way, that a pressure gradient arises which initiates a fluid flow from the left to the right. In addition, a time-constant Dirichlet boundary condition for concentration c (x = 0), i.e. $\bar{c} = 0.1$ is prescribed. Within the domain an initial condition is used for the concentration, setting $c_0(x) = 0.001$. Thus the porous medium is already saturated initially with a suspension. All four constituents are therefore present. The consequence of the present initial and boundary conditions is a concentration influx leading to concentration front passing through the domain from left to right. The boundary and initial conditions as described above are summarized in Fig. 2a). The time-scale of the simulation covers the duration of the concentration front propagation through the whole domain.



Figure 2: Infiltration process: a) initial and boundary conditions for the investigated domain; b) illustration of the concentration profile for $t > t_0$

According to Fig. 3 the porosity $\phi(x, t)$ is changing in space and time. Hence, parallel to the convective transport, an infiltration process takes place leading

to a decrease of fluidized particles. This also indicates the smooth, gradual character of the infiltration process. The values describing the concentration c and the filter velocity of the fluid q are at their maximum in the begin of the process. Although the Darcy velocity q is influenced by the increase in viscosity, Eq. (24), and a decrease in permeability, Eq. (23), the latter physical phenomenon is dominating.



Figure 3: Contour plot describing the evolution of the porosity in space and time in the domain

In contrast to classical seepage flow in porous media, the filter velocity \boldsymbol{q} of the front depends on the effective velocity which is a function of the concentration of fines. Furthermore, the permeability is also not constant in the domain and depends on the amount of infiltrated and deposited particles. Within the flow-through fine grained material is deposited. Furthermore, it is observed that the transport processes are essentially driven by convection. Diffusion processes play only a minor role. This is concluded from the moving front (steep gradient of concentration).

Next, we discuss the evolution of concentration in detail. It has to be noted, that in the case $t_{eq} > t_0$ an equilibrium concentration c_{eq} is reached. Microscopically, this process can be explained as follows: The concentration in a certain control volume is not changed while the local deposition is ongoing and the local permeability $k^{\mathfrak{s}}$ is decreasing. The transport of the fine particle fraction of the domain causes a sudden change of the concentration in the transition zone. This sudden decrease of the concentration disappears as soon

Length of domain	L	20	m
Pressure at boundary $x = 0$	\bar{p}_0	6.38	kPa
Pressure at boundary $x = L$	\bar{p}_L	0	Pa
Concentration at boundary $x = 0$	\bar{c}	0.1	
Effective dynamic viscosity	$\eta^{\mathfrak{f}R}$	1	mPas
Initial porosity	ϕ_0	0.32	
Initial intrinsic permeability	$k_0^{\mathfrak{s}}$	8.57×10^{-10}	m^2
calculation time	T	1×10^5	\mathbf{S}

Table 1: Material properties and used boundary conditions

as the transition zone has passed the whole domain and arrived at the right boundary. If the infiltration process is considered beyond the flow-through of the transition zone, an equilibrium concentration setting is observed in the whole domain. In particular, the chosen parameter k in the evolution equation of the volume exchange term, has an influence on the time-independent equilibrium concentration. In Fig. 4 equilibrium concentrations of fines in the suspension for different values of k are illustrated, whereas the remaining simulation details are unchanged. It is observed that for sufficiently small values of k ($10^{-4} < k < 0.02$) the concentration profile across the domain shows an approximately linear behaviour. As a result, the concentration change is constant between two time steps in the same location for small k. The scatter plot of the concentration distribution can be represented by a linear function. These resulting functions, describing the distribution of concentration c_{eq} in the domain are shown in Fig. 4 b). The conclusion is, that as long as values of k are considered, which lead to a linear distribution of equilibrium concentration, the results for different parameters of k can be converted into each other. When a distribution of equilibrium concentration is known for a certain factor k_M , it is possible to derive the distribution of concentration for another value of k. This means a master curve for the distribution of equilibrium concentration can be determined. Accordingly, the transformation follows the equation

$$c_{eq}\left(\frac{x}{L}\right) = m_M \frac{k}{k_M} \frac{x}{L} + \bar{c}.$$
(27)

In Eq. (27), m_M describes the slope of the concentration distribution, arising when k_M is used. The resulting concentration distribution is valid for one value of k. \bar{c} is the previously mentioned boundary condition for the con-



Figure 4: Distribution of the concentration c_{eq} at equilibrium, left: calculated concentration distribution, right: fitted and transformed concentration distribution

centration. Therefore, the graph for $k_M = 0.02$ illustrates the master curve. This leads to a slope of $m_M = -0.03415$ and $\bar{c} = 0.1$. The resulting distribution of concentration using these values and Eq. (27) are shown in Fig. 4 as solid lines. This fits very well with the calculated concentration gradients.

For larger values of k it is not sufficient to reduce the transformation of the concentration distribution to a pure rotation. This results from the non-linearity of the gradient of concentration.

Considering the results of the presented analyses, it is point out, that the proposed model is well-suited for the simulation of a filter cake formation. A filter cake is a localized layer of highly reduced permeability. For an explanation of this term a 1-dim domain is divided into two parts. In the left part of the domain the volume fraction of the solid skeleton is assumed to be zero $n^{\mathfrak{sn}} = 0$. The right part is represented by a four-phase mixture in accordance to the parameters used in the analysis. If an IBVP is solved with the previously described boundary conditions, two different types of filter cakes might occur. First, it is possible that an external filter cake is formed, cf. Fig. 5. Thereby the constriction sizes of the pore channels are smaller than the average diameter of the fluidized fines. Hence, the fines are not able to penetrate significantly in the second part of the domain. Instead the fines are deposited on the border in between the both parts.

As a result, more particles accumulate at the interface, so that a layer with reduced permeability can be observed.



Figure 5: Formation of a filter cake. Left: external filter cake; right: internal filter cake

In contrast, the formation of an internal filter cake occurs cf. Fig. 5, if fines penetrate over a certain distance into the second part of the domain. The infiltration causes a local increase in the flow rate q and a reduction of the permeability $k^{\mathfrak{s}}$. Hence the production term $\hat{n}^{\mathfrak{a}}$ is increased locally. As a result, a high amount of fines is infiltrated locally. It has to be pointed out, that the proposed model is applicable to cases of internal filter cakes only. To capture the occurrence of an external filter cake geometrical consideration concerning the analysis of the constriction size distribution are necessary.

5 Conclusion

In the preceding sections the governing differential equations of internal erosion processes of cohesionless soils were formulated based on the mass and momentum balances of the present phases. In addition material incompressibility was assumed. Within this framework, a four-phase model for infiltration processes was presented.

Furthermore, the IBVP was extended by a production term to simulate the infiltration of fines from the suspension to the solid fraction. The form of the introduced production term $\hat{n}^{\mathfrak{a}}$ can be understood as a constitutive formulation. The production term is thermodynamically consistent and contains a material parameter k, which has to be validated by physical experiments.

However, for a given combination of a filter material and a suspension, the appropriate values of k have to be determined by conducting infiltration experiments. For small values of k (representing linear profiles of the equilibrium concentration c_{eq}) a master curve was found, which could be used to avoid experimental set-ups for respective soil materials. In particular, for larger values of k, resulting in a nonlinear concentration distribution, further theoretical, numerical and experimental investigations are necessary. In the future, additional investigations will be carried out with the aim to take the pore network into account and to describe its effects on the rearrangement phenomena in porous media. This will lead to an even more sophisticated description of the microscopical infiltration process.

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