Multiphase FEM Modeling of Infiltration Processes in Cohesionless Soils

Alexander Schaufler[1], Christian Becker[1], Holger Steeb[1]

[1] *Mechanics–Continuum Mechanics, Ruhr-University Bochum, Germany*

Adress all correspondence to Alexander Schaufler, E-mail: schaufler@lkm.rub.de

Originally published in

Proc. Appl. Math. Mech. [2012], **12 (1)**, pp 389–390.

Abstract

The aim of this work is to develop a continuum multiphase model to describe infiltration processes for cohesionless soils. For this purpose, a Representative Volume Element (RVE) is considered and described by the continuum mixture theory extended by the concept of volume fractions (Theory of Porous Media - TPM). The thermodynamically consistent TPM is a macroscopical multiphase modeling approach, extended from classical single-phase continuum mechanics. Futhermore a 1-dim example for an infiltration problem is presented.

1 Introduction

The application of a macroscopical hydraulic gradient to a fluid saturated cohesionless soil causes seepage flow. Furthermore, the micro-structure of the porous skeleton dominates the physics of infiltration processes of complex fluids (fluid & fines) and thus the evolution of the hydraulic and mechanical properties of the soil. From a modeling point of view, approaches taking into account the macro- and the micro-scale physics are yet not well established. The transportation process of fines through the pore network strongly depends a) on the mentioned hydraulic boundary conditions and b) on the microscopic topology of the pore space.

In addition to the transport of fine particles, also the infiltration, i.e. the attachment of fine particles to the coarse-grained solid skeleton, is crucial for the evolution of the hydraulic properties in the volume. The resulting numerical model is applied to different infiltration applications. One specific application of the proposed model is situated in mechanized tunneling. In that case, the infiltration of grout in the surrounding soil leads to the evolution of hydraulic properties of the soil and is finally responsible for deformations on the surface.

2 Continuum modeling of infiltration processes

The TPM is used to describe the transport and the infiltration of fluidized particles in a liquid through a porous medium. First, the fully saturated RVE is divided into four constituents φ^α with $\alpha = \{\mathfrak{f}, \mathfrak{a}, \mathfrak{sn}, \mathfrak{sa}\}$, which are described by their volume fractions $n^\alpha = \mathrm{d}v^\alpha/\mathrm{d}v$ Then, the mass balances of the accordant constituents and the mixture are evaluated to describe the transport process. Following de Boer [1], Ehlers & Bluhm [2], and Steeb [4] the general partial mass balance in local form can be written as

$$(\rho^\alpha)'_\alpha + \rho^\alpha \operatorname{div} \mathbf{v}_\alpha = \hat{\rho}^\alpha =: \hat{n}^\alpha \rho^{\alpha R}. \tag{1}$$

Here, ρ^α is the partial density, $\mathbf{v}_\alpha$ the velocity, and $\hat{\rho}^\alpha$ the density production rate of the constituent φ^α. Moreover, the infiltration of the fine components $\varphi^\mathfrak{a}$ is realized by a mass exchange term $\hat{n}^\mathfrak{a}$

$$\hat{n}^\mathfrak{a} = -k\, c\, |\mathbf{q}|\,, \tag{2}$$

in case of material incompressible constituents $\rho^{\alpha R} = \rho_0^{\alpha R}$. This can be understood as a constitutive assumption, k is a material parameter and c the concentration of fines in the suspension. For this example it is assumed that the mass exchange occurs only within two constituents $\hat{n}^{\mathfrak{a}} = -\hat{n}^{\mathfrak{s}\mathfrak{a}}$. Note, that for a more detailed derivation of the governing equations we refer to a previous publication [3]. Thus, the following set of equations is formulating the Initially Boundary Value Problem (IBVP) of infiltration:

$$
\begin{aligned}
\operatorname{div}\left[\frac{k^{\mathfrak{s}}(\phi)}{\eta^{\mathfrak{f}}(c)}\operatorname{grad} p\right] &= 0, \quad \forall \mathbf{x} \in \mathcal{B} \times T, \\
\partial_t(c\,\phi) + \operatorname{div}\left[c\,\frac{k^{\mathfrak{s}}(\phi)}{\eta^{\mathfrak{f}}(c)}\operatorname{grad} p\right] &= \hat{n}^{\mathfrak{a}}, \quad \forall \mathbf{x} \in \mathcal{B} \times T,
\end{aligned}
$$

where $k^{\mathfrak{s}}$ is the intrinsic permeability, ϕ the porosity, $\eta^{\mathfrak{f}}$ the viscosity of the suspension, and p the pressure, with boundary conditions for the flux $\mathbf{q}$ at the Neumann boundary Γ_N and the pressure p at the Dirichlet boundary Γ_D

$$
q = \mathbf{q}\cdot\mathbf{n} = \overline{q}, \quad \forall \mathbf{x} \in \Gamma_N \times T, \qquad \text{and} \qquad p = \overline{p} \,\wedge\, c = \overline{c}, \quad \forall \mathbf{x} \in \Gamma_D \times T.
$$

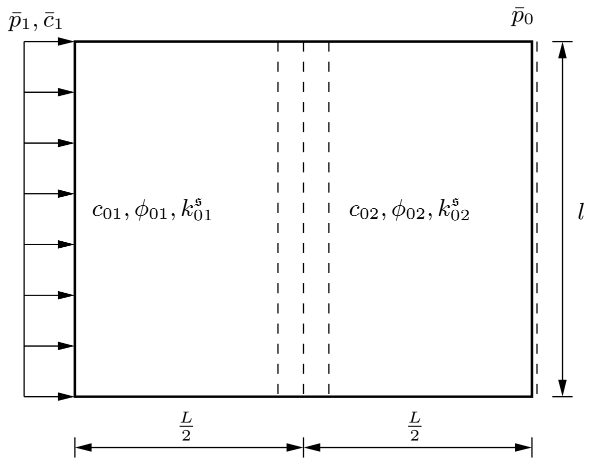

Figure 1: Initial and boundary conditions for the IBVP.

3 Discussion

Using the IBVP (Fig. **??**), described in the last section, a 1-dim infiltration problem was calculated. A (physically 1-dim) domain with the aspect ratio

$L/l = 20$ was considered numerically. On the left edge, a concentration of fines of $\bar{c}_1 = 0.1$ and a pressure $\bar{p}_1 = 6.38\,\mathrm{kPa}$ were applied. On the right edge, a free boundary was simulated, which corresponds to a constant pressure $\bar{p}_0 = 0\,\mathrm{kPa}$. Furthermore, the concentrations of fines were set to $c_{01} = c_{02} = 0.01$. In the left part of the domain a porosity $\phi_{01} = 0.32$ and in the right part $\phi_{02} = 0.45$ were applied. Accordingly, following the Kozeny-Carman equation, an initial permeability $k^{\mathfrak{s}}_{01} = 8.58 \cdot 10^{-10}\,\mathrm{m}^2$ in the left part and $k^{\mathfrak{s}}_{02} = 3.65 \cdot 10^{-9}\,\mathrm{m}^2$ in the right part of the domain were enforced. To avoid a jump in the porosity distribution a step function between the different porosity values was created.

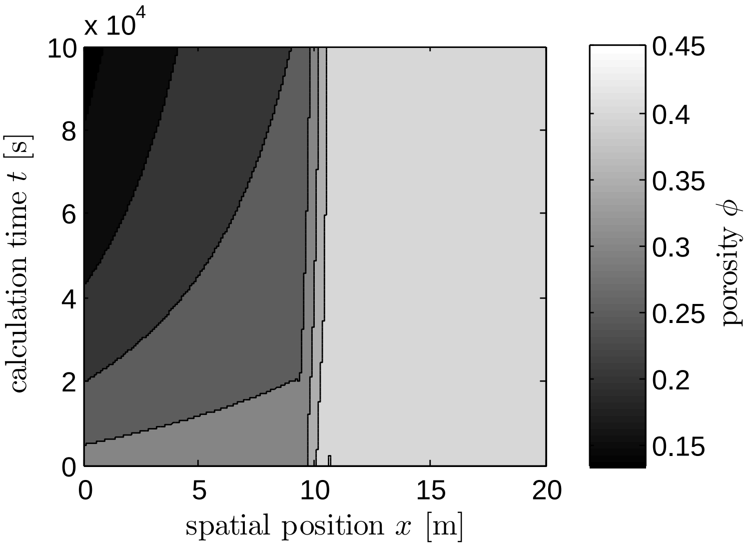

Figure 2: Contour plot describing the evolution of porosity in space and time in the domain.

In Fig. **??**, the results for the porosity distribution in space and time are plotted. In the left part a continuous infiltration process takes place, leading to a steady decrease of the porosity. Due to the fact that the number of fluidized particles decreases through infiltration by attachment of particles to the solid skeleton, the maximum change of porosity takes place in the spatial position where the highest concentration of fines is located.

From the left edge to the middle of the domain the amount of fluidized particles decreases and thus also the change of the porosity. Behind the transition zone where the two porosities are adapted to each other by a step function, the porosity remains nearly constant in the right area of the domain. On closer analysis, a decrease in porosity can be determined. However, the change of the porosity is of smaller magnitude than in the left area of the domain. The reason for this approximately constant porosity distribution in the right part of the domain is that, due to the lower initial porosity in the left area, the major part of the particles are already attached to the solid skeleton in the left part of the domain. The particles, which are not blocked by the area with lower initial porosity, can pass the area with the porosity ϕ_{01} without infiltration.

4 Conclusion

A flow of a suspension through a porous medium was described by evaluation of the partial mass balances as also the mass of the mixture. To capture the infiltration process of the fluidized particles to the solid skeleton a mass production term $\hat{n}^{\mathfrak{a}}$ was added to take phase transition into account. A 1-dim numerical example was presented, which leads to a better understanding of clogging phenomena during infiltration processes, e.g. the formation of a filter cake.

Acknowledgement

Financial support was provided by the German Science Foundation (DFG) in the framework of project B4 of the Collaborative Research Center SFB 837. This support is gratefully acknowledged.

References

[1] R. de Boer, Theory of Porous Media: Highlights in Historical Development and Current State (Springer, 2000).

[2] W. Ehlers and J. Bluhm, Porous media: theory, experiments, and numerical applications (Springer, 2002).

[3] A. Schaufler, C. Becker, and H. Steeb, Infiltration processes in cohesionless soils, Zeitschrift für Angewandte Mathematik und Mechanik, 2012, *(under review)*.

[4] H. Steeb, Non-Equilibrium Processes in Porous Media, Habilitation, Universität des Saarlands, Saarbrücken, 2008.